Program

14.06			
8.00-8.45	Registration		
8.45-9.00	Opening		
9.00-9.30	Hietarinta	15.00-15.30	Ramani
9.30-10.00	Viallet	15.30-16.00	Kozlov
10.00-10.30	Sergeev	16.00-16.30	Boelen
Coffee break		Coffee break	
11.00-11.30	Scimiterna	17.00-17.30	Sasaki
11.30-12.00	Nikolov	17.30-18.00	Kounchev
15.06			
9.00-9.30	Ormerod	15.00-15.30	Hone
9.30-10.00	Yang Shi	15.30-16.00	Zwicknagl
10.00-10.30	Goryuchkina	16.00-16.30	Kassotakis
Coffee break		Coffee break	
11.00-11.30	Satsuma	17.00-17.30	Iliev
11.30-12.00	N. Joshi	17.30-18.00	Dzhamay
12.00-12.30	Gaic	18.00-18.30	
16.06			
9.00-9.30	Winternitz	Free	Afternoon
9.30-10.00	Kajiwara		
10.00-10.30	Dinh Tran		
Coffee break			
11.00-11.30	M. Visinescu	Free	Afternoon
11.30-12.00	B. Nteumagne		
12.00-12.30	Doliwa		
17.06			
9.00-9.30	Yakimov	15.00-15.30	Levi
9.30-10.00	Papageorgiu	15.30-16.00	Mikhailov
10.00-10.30	Kouloukas	16.00-16.30	Wang
Coffee break		Coffee break	
11.00-11.30	Lobb	17.00-17.30	Fordy
11.30-12.00	Xenitides	17.30-18.00	Verge-Rebelo
12.00-12.30	Suris	18.00-18.30	
18.06			
9.00-9.30	Spicer	15.00-15.30	Dryuma
9.30-10.00	Hay	15.30-16.00	Atkinson
10.00-10.30	Nieszporski	16.00-16.30	A. Visinescu
Coffee break		Coffee break	
11.00-11.30	Pelloni	17.00-17.30	Gerdjikov
11.30-12.00	Aneva	17.30-18.00	Grahovski
12.00-12.30		18.00-18.30	Kostov

Random walk polynomials and models of nonequilibrium physics

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We consider the open asymmetric exclusion process which is the fundamental model of non-equilibrium physics with a wide range of applications, like biological transport, traffic flow, etc. We study the model within an algebraic approach based on the symmetry properties of the process. The boundary symmetry is generated by two nonlocal charges defined by the boundary conditions of the process. The infinite-dimensional representations of the boundary algebra in terms of the Askey-Wilson polynomials are important for the exact solvability in the stationary state. One of the boundary generators is identified with the second order difference operator for the Askey-Wilson polynomials and can be interpreted as the transition rate matrix of the process. The Askey-Wilson polynomials have the properties of random walk polynomials for the model.

Q4 Solitons

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I will describe a new technique of solving the Backlund iteration scheme and thus obtaining the soliton-type solutions of discrete KdV-type equations. The focus will be on a newly obtained explicit N-soliton solution of the primary model in the Adler-Bobenko-Suris exhaustive list of canonical forms for these systems, namely the equation Q4.

Orthogonal polynomials and discrete Painlevé equations

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We look at how how the recurrence coefficients of certain semiclassical orthogonal polynomials solve discrete Painlevé equations. We describe how these solutions can be computed by using the fact that they are, in some sense, unique with respect to boundary conditions which arise naturally in the orthogonal polynomial context.

On Non-Symmetric Discrete Toda Systems and Their Relation to Quad-Graphs (poster)

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We establish a relation of non-symmetric discrete relativistic Toda type equations to 3D consistent systems of quad-equations. Moreover, we will present a master equation which includes both symmetric and non-symmetric discrete relativistic Toda type equations as particular or limiting cases. In addition, we will give a survey at adequate derivations of non-symmetric discrete relativistic Toda type equations from this master equation. Our construction allows for an algorithmic derivation of the zero curvature representations and yields analogous results for the continuous time case.

The affine Weyl group symmetry of Desargues maps and quadrilateral lattices

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I will present new incidence geometric description of the (non-commutative) Hirota-Miwa system in terms of the so-called Desargues maps, and its equivalence to the quadrilateral

lattice theory. The symmetry of the Desargues theorem, which is the cornerstone of the multidimensional compatibility of the Desargues maps naturally brings to light the A-type affine Weyl group symmetry of the system.

Hirota-Kimura type discretization of the classical nonholonomic Suslov problem

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We constructed Hirota-Kimura type discretization of the classical nonholonomic Suslov problem of motion of rigid body fixed at a point. We found a first integral proving integrability. Also, we have shown that discrete trajectories asymptotically tend to a line of discrete analogies of so-called steady-state rotations. The last property completely corresponds to well-known property of the continuous Suslov case. The explicit formulae for solutions are given. In n-dimensional case we give discrete equations.

Multidimensional Plebanski metrics and their properties

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Particular solutions of nonlinear differential equations defined by the Ricci-flat 6D and 8D Plebanski metrics are constructed. Their properties are discussed.

Properties of the Beltrami-Laplace operators of the Plebanski 6D and 8D metrics are studied.

Special Lagrangian equations

$$\Delta u = \det(D^2 u)$$

having importance in the study of string theory are derived and their particular solutions are constructed.

References

[1] V.Dryuma, Multidimensional the Ricci-flat spaces defined by nonlinear equations. arXiv: 0911.2799 v1 [physics.gen-ph] 14 Nov, 2009, 1–12.

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[3] V.S. Dryuma, The Riemann Extension in theory of differential equations and their applications, *Matematicheskaya fizika, analiz, geometriya*, Kharkov, 2003, v.10, No.3, 1–19.

On Geometric Configurations Related to Matrix Factorizations

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An important example of a discrete Lax-Pair representation of an integrable system is a re-factorization transformation of its Lax matrix. We study rational Lax matrices represented as products of Blaschke-Potapov factors. Using residue techniques we show that the exchange rules for the components of such factors can be visualized by labeled atomic trivalent graphs, and that such triples can be glued together following certain strict rules. In particular, the configuration corresponding to the transposition of two such blocks is a cube and both the correct choice of the Lagrangian coordinates and the expression for the Lagrangian itself can be readily obtained from the labeling of the edges and the vertices of this cube.

Integrable maps and Poisson algebras derived from cluster algebras

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We consider a class of map, recently derived in the context of cluster mutation [3]. Not all of these are integrable, but we specialise to some particular families which are. We discuss invariant Poisson brackets, along with the Poisson algebra of special families of functions associated with some of these maps. For one particular family, a bi-Hamiltonian structure is derived and used to construct a sequence of Poisson commuting functions and hence show complete integrability [1].

One of the most important open questions regards the identification and classification of the integrable cases of maps obtained through the construction of [3]. This classification is the main theme of [2].

References

[1] A.P. Fordy. Mutation-periodic quivers, integrable maps and associated Poisson algebras. 2010. Preprint arXiv:1003.3952v1 [nlin.SI].

[2] A.P. Fordy and A.N.W. Hone. Integrable maps and Poisson algebras derived from cluster algebras. 2010. In preparation.

[3] A.P. Fordy and R.J. Marsh. Cluster mutation-periodic quivers and associated laurent sequences. 2009. Preprint arXiv:0904.0200v2 [math.CO].

On Discrete evolution equations related to simple Lie groups

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The Ablowitz-Ladik approach to integrate discrete evolution equations is related to the simple Lie group $SL(2)$. By using proper parametrization of the Lax operator L_n we generalize it to simple Lie groups of higher rank. Important particular cases of multicomponent DEE are related to symmetric spaces and provide integrable discretizations of multicomponent nonlinear Schrödinger equations.

Asymptotic forms and asymptotic expansions of solutions to Painleve equations

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We consider Painlevé equations [1] which are nonlinear ordinary differential equations of the second order, and their solutions determine new functions (Painlevé transcendents). For these equations we seek their formal solutions, which are represented in asymptotic expansions close to a point $x = x_0$ (including zero and infinity) of form

$$y = c_r(x - x_0)^r + \sum_s c_s(x - x_0)^s,\ s \in \mathbf{K}, \tag{1}$$

of five types: the power ([2],§3), the power-logarithmic ([2], §3), the complicated, the exotic and the half-exotic [3]. In addition we can calculate exponential asymptotic forms ([2], §5) and exponentially small additions ([2], §7) to the expansions (1).

Besides, we proved the theorem on convergence of formal solution of nonlinear ordinary differential equation $f(x, y) = 0$ of the order n ([3], Ch. 1, §7), where f is a differential sum ([2], §1).

We found all expansions (1) of the five types near three singular and non-singular points of the sixth Painlevé equation for all values of its four complex parameters.

For the formal solutions to the first four Painlevé equations there are also exist elliptic and exponential asymptotic forms and exponentially small additions to the expansions (1).

References

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The Camassa-Holm Hierarchy and Soliton Perturbations

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A brief survey of the theory of soliton perturbations is presented. The focus is on the usefulness of the so-called Generalised Fourier Transform (GFT). This is a method that involves expansions over the complete basis of “squared solutions” of the spectral problem, associated to the soliton equation. The Inverse Scattering Transform for the corresponding hierarchy of soliton equations can be viewed as a GFT where the expansions of the solutions have generalised Fourier coefficients given by the scattering data.

The GFT provides a natural setting for the analysis of small perturbations to an integrable equation: starting from a purely soliton solution one can ’modify’ the soliton parameters such as to incorporate the changes caused by the perturbation.

As an illustrative example, the perturbation theory for the Camassa-Holm hierarchy is presented.

References

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Casorati solutions to non-autonomous DSKP and DSKdV equations

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We use known solutions to bilinear equations to solve other well-known integrable nonlinear partial difference equations.

Beginning with non-autonomous Casorati determinant solutions to the discrete two-dimensional Toda lattice equation, explicit solutions to the discrete Schwarzian KP equation and the non-autonomous DSKdV (or cross-ratio) equation are constructed. This process requires a reduction, which is problematic because the DSKdV equation is non-autonomous. The problem is solved by allowing one dimension to be autonomous, taking on an auxiliary role, and making the reduction there.

Hirota's method and the three-soliton condition for difference equations

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Integrable PDEs are characterized by many interesting properties, one of which is the existence of multi-soliton solutions. The construction of these solutions is particularly simple using Hirota's direct method. It turns out that although many equations have one and

even two-soliton solutions, the existence of three-soliton solutions is in practice equivalent to integrability. This provides a method for searching for integrable equations. We briefly review the situation in the continuum case and then present some results in the discrete case.

Symplectic structure of cluster maps associated with quivers

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A quiver is a graph with arrows along its edges. Associated with any quiver without 1- or 2-cycles is a set of maps that generate a cluster algebra via mutation, in the sense introduced by Fomin and Zelevinsky. Fordy and Marsh recently classified all such quivers that are periodic under mutation, with period 1, meaning that the map corresponding to mutation at one of the vertices can be realized by iteration of a single recurrence relation. Here it is shown that in the latter situation there is always an associated birational map that is symplectic. As a particular example of this construction, it is explained how to derive a Poisson structure for reductions of the Hirota-Miwa equation. (This is joint work with Allan Fordy.)

Bispectrality of multivariable orthogonal polynomials

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It is well known that polynomials orthogonal on the real line $p_n(x)$ satisfy a three term recurrence relation (i.e. they are eigenfunctions of a second-order difference operator acting on n). The polynomials which are also eigenfunctions of a second-order differential or difference operator acting on x have a long history and many applications. They can be obtained as reductions or limits of the Askey-Wilson polynomials. I will discuss classification results and bispectral properties for multivariable orthogonal polynomials.

Can we use geometry to find properties of solutions of discrete Painleve equations?

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Integrable non-QRT mappings of the plane

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We construct 9-parameter and 13-parameter dynamical systems of the plane which map bi-quadratic curves to other bi-quadratic curves and return to the original curve after two iterations. These generalize the QRT maps which map each such curve to itself.

New integrable systems of interacting nonlinear waves (discrete type Hamiltonian systems)

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We obtain two new completely integrable Hamiltonian systems (discrete Hamiltonian systems) of the following type

$$i\frac{d\vec{u}}{dt} = \vec{F}(\vec{u})$$

with cubic nonlinearity, which have the Lax representation and appropriate set of first integrals. Thus we extend the results of [1]. One of these systems in the two dimensional case, after appropriate reduction contains the so called dimer equation and the last equation is investigated using the finite-gap integration method.

References

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Poisson Yang-Baxter maps and integrability

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A family of multidimensional Poisson Yang-Baxter maps is presented. They admit zero curvature representation in terms of polynomial matrices. Multidimensional integrable mappings arise by considering periodic initial value problems on quadrilateral lattices.

Poisson Yang-Baxter maps and integrability

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In a recent research we have developed a new point of view on the multivariate Moment problem, by introducing the so-called pseudopositive measures, cf. [1], [2]. This brings into game new approach to multivariate orthogonality which is interesting as a source of new integrable models, and new isospectral deformations.

References

[1] O. Kounchev, H. Render (2010) The moment problem for pseudo-positive definite functionals, Arkiv for Matematik, 48 :97-120.

[2] O. Kounchev, H. Render, Reconsideration of the multivariate moment problem and a new method for approximating multivariate integrals, **arXiv:math/0509380.**

Hamiltonian formalism for discrete equations. Symmetries and first integrals.

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We consider the relation between symmetries and first integrals for discrete Hamiltonian equations. The results are built on those for canonical Hamiltonian equations. The canonical Hamiltonian equations can be obtained by variational principle from an action functional. We develop an analog of the well-known Noether's identity for canonical Hamiltonian equations and their discrete counterparts (discrete Hamiltonian equations). The approach based on symmetries of the discrete action functional provides a simple and clear way to construct first integrals of discrete Hamiltonian equations just by means of algebraic manipulations. It can be used to conserve structural properties of underlying differential equations under discretization that is useful for numerical implementation. The results are illustrated by a number of examples.

Generalized symmetry integrability test of discrete equations on the square lattice

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Imposing the existence of a generalized symmetry for equations defined on a square lattice we obtain an integrability test. We use this test to classify some nonlinear affine equations which pass the multiple scale A_3 test.

λ-symmetries for discrete equations (poster)

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We define as λ-symmetries for differential equations a subclass of the potential symmetries where the symmetry generator do not depend on the potentials. This definition can be extended to the case of difference equations and we can apply it to some nontrivial examples.

Lagrangian formulation of multidimensionally consistent equations I: Difference equations

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Multidimensional consistency is a key integrability property of certain discrete equations; it implies that we are dealing with infinite hierarchies of compatible equations rather than with one single equation. Requiring this property to be reflected also in the Lagrangian formulation of such equations, we arrive naturally at the construction of Lagrangian multiforms, i.e. Lagrangians which are the components of a form and satisfy a closure relation. From these Lagrangian multiforms we can then derive any equation in the hierarchy. On the basis of this we propose a new variational principle for integrable systems which brings in the geometry of the space of independent variables.

Canonical conservation laws and integrability conditions for difference equations

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In this work we try to give a consistent background and definitions suitable for the theory of integrable difference equations. We adapt a concept of recursion operator to difference equations and show that it generates an infinite sequence of symmetries and canonical conservation laws for a difference equation. Similar to the case of partial differential equations these canonical densities can serve as integrability conditions for difference equations.

Vector-decomposition of a finite rotation (poster)

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The presentation of rigid body displacements is an old mechanical problem but of a great importance in solving different real tasks. In geometrical and computational mechanics rigid body kinematics is considered through vectors and matrices approaches. In this aspect the rotation group in three dimensional space may be described combining our knowledge from analytical mechanics, vector analysis, algebra and differential geometry. Here is the place to mention that the different parameterizations of the rotation group $SO(3)$ influence on the efficiency of the kinematic and dynamic models as at one rigid body so in multibody mechanical systems. The analytical representations of any rotation can be expressed by defining its action on vectors, quaternions or spinors. On the other hand the parameterizations of the rotations are: Eulerian angles in the classical 3-1-3 sense and all other combinatons like: 3-2-3, 3-2-1, 1-2-3 and etc., are known as Bryant angles, Eulerian parameters, Cayley-Klein parameters and etc.

To find the resultant axis and angle of rotation after two, three or more finite partial rotations is a problem very important in multibody mechanics. But the inverse problem, namely, to decompose a finite rotation into three partial rotations about prescribed axis is a more difficult one and it is very important in motion planning in the group of rotations and inverse kinematic problem at a manipulator system as well. The present paper gives explicit formulae in solving this problem using vector-like parametrization of rotation group. In the first part the notion vector-parameter is introduced and its properties are given. After the problem is solved a numerical example is given. The method is realized analytically using the computer algebra system Mathematica.

Consistency on cubic lattices for determinants of arbitrary orders

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We consider a special class of two-dimensional discrete equations defined by relations on elementary $N \times N$ squares, $N > 2$, of the square lattice $\mathbb{Z}^2$, and propose a new type of consistency conditions on cubic lattices for such discrete equations that is connected to bending elementary $N \times N$ squares, $N > 2$, in the cubic lattice $\mathbb{Z}^3$. For an arbitrary N we prove such consistency on cubic lattices for two-dimensional discrete equations defined by the condition that the determinants of values of the field at the points of the square lattice $\mathbb{Z}^2$ that are contained in elementary $N \times N$ squares vanish.

Acknowledgements. This research was supported by the Russian Foundation for Basic Research (project no. 09-01-00762) and the Program for Support of Leading Scientific Schools (project no. NSh-1824.2008.1).

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Symmetry and dimension reductions

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NA

Discrete Weingarten congruences revisited.

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We review results on integrable discretization of Weingarten rectilinear congruences.

First Integrals and Conservation Laws of Difference Equations

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We determine first integrals of difference equations by use of their continuous symmetries. We apply the technique of finding first integrals of differential equations to the discrete case. A review on conservation laws of difference equations is also provided.

Symmetries of the associated linear problems for q-Painleve equations.

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We will consider a group of symmetries of the associated linear problems for some q-Painleve equations. The symmetries, as does the q-Painleve equation itself, may be expressed in terms of connection preserving deformations. We consider two cases of this theory as examples, namely the associated linear problems for q-PII and q-PV.

The elliptic sine-Gordon equation

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The spectral theory of the elliptic sine-Gordon equation posed in various domains will be presented, comparing the results with the corresponding results for the time-dependent, evolution models.

On a difference analog of Holm-Pavlov type hydrodynamical systems and their integrability

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Based on the gradient-holonomic algorithm we analyze the integrability property of a generalized discrete Holm-Pavlov type hydrodynamical equations. The infinite hierarchies of polynomial and non-polynomial conservation laws, both dispersive and dispersionless are constructed. Special attention is paid to the cases $N = 2$, 3 and $N = 4$, for which the conservation laws, Lax type representations and bi-Hamiltonian structures are analyzed in detail.

Generalizing the QRT mapping

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We present an extension of the QRT mapping beyond the familiar symmetric and asymmetric varieties. Starting from our results on discrete Painleve equations, we show that there exist integrable QRT-like mappings, the coefficients of which are periodic functions. We present several examples of mappings with periodic coefficients of various periods and show that there exist cases where the periods are arbitrarily long. We prove the integrability of all the examples by constructing the corresponding conserved quantities and we show how these systems, just as their QRT siblings, can be explicitly integrated in terms of elliptic functions.

On the definition of finite differences approximating partial derivatives on non orthogonal lattices

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Definitions of finite differences approximating partial derivatives on potentially non orthogonal lattices are given. Those definitions are then used as new variables in the discrete space. They give rise to new formulas for the invariants of the infinitesimal generators which are such that the symmetry preserving discretization of PDEs becomes straightforward. An example is given.

An integrability test based on multiscale analysis: classification of integrable, dispersive and affine linear discrete systems on the square.

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We provide a set of integrability conditions for a class of discrete systems defined on the square.

Exceptional Orthogonal Polynomials and difference Schrödinger equations

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Orthogonal polynomials satisfying second order differential or difference equations are severely limited by Bochner's theorem and its difference version. Recently, several series of infinitely many exceptional orthogonal (Laguerre, Jacobi, Wilson and Askey-Wilson) polynomials were constructed by Odake-Sasaki as exact solutions of the ordinary and difference

Schrödinger equation in one dimension. These polynomials are exceptional in the sense that they start from degree $\ell \geq 1$ and thus not constrained by any generalisation of Bochner's theorem.

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Quantum Q4

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I will present a new master solution of quantum star-triangle (Yang-Baxer) equation which contains as special cases all continuous and discrete spin solutions that were previously known. In the classical and a specific hybrid classical/discrete spin limits the leading singular term of the master quantum 2D lattice model is the Adler-Bobenko-Suris Q4 Laplace-type system on quad-graphs.

Higher analogues of the discrete-time Toda equation and the quotient-difference algorithm

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The discrete-time Toda equation arises as a universal equation for the the relevant Hankel determinants associated with one-variable orthogonal polynomials through the mechanism of adjacency, which amounts to the inclusion of shifted weight functions in the orthogonality condition. In this paper we extend this mechanism to a new class of two-variable orthogonal polynomials where the variables are related via an elliptic curve. This leads to an 11-point quadrilinear equation for the associated Hankel determinants, together with its Lax pair, which is derived from the relevant recurrence relations for the orthogonal polynomials. In a similar way as the quotient-difference (QD) algorithm is related to the discrete-time Toda

equation, a novel quotient-quotient-difference (QQD) scheme is presented for the 11-point quadrilinear equation.

On the Lagrangian structure of integrable quad-equations

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The new idea of flip invariance of action functionals in multidimensional lattices was recently highlighted as a key feature of discrete integrable systems. Flip invariance was proved for several particular cases of integrable quad-equations by Bazhanov, Mangazeev and Sergeev and by Lobb and Nijhoff. We provide a simple and case-independent proof for all integrable quad-equations. Moreover, we find a new relation for Lagrangians within one elementary quadrilateral which seems to be a fundamental building block of the various versions of flip invariance. (Joint work with)

Sufficient number of integrals for the p-th order Lyness equation

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We present a sufficient number of explicit integrals for the Lyness equation of arbitrary order.We use the staircase method to construct integrals of a derivative equation of the Lyness equation. Closed-form expressions for the integrals are given based on a non-commutative Vieta expansion.The integrals of the Lyness equation follow directly from these integrals.Previously found integrals for the Lyness equation arise as special cases of our new set of integrals.

Which invariants for discrete integrable systems?

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The vanishing of algebraic entropy is the sign of integrability for maps and lattice maps. It means that the degree of the iterates of a the map grows polynomially rather that exponentially. A second characteristic is then the rate of this polynomial growth. Linear growth has to do with linearizability and is regarded as trivial. Quadratic growth is standard for "classical" models like QRT or discrete Painleve. What about higher rates? We will relate this question to the type of invariants we are prepared to accept as integrals of motion. In other words: where do we put the the frontier of integrability?

Periodic and solitary wave solutions of two-component Zakharov-Yajima-Oikawa system using Madelung's approach

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A two component one-dimensional long wave - short wave resonant system (Zakharov-Yajima-Oikawa completely integrable [1],[2],[3]) was discussed using Madelungs approach [4], [5]. For motion with stationary profile current velocities the periodic solutions, expressed through Jacobi elliptic functions, and the solitary wave solutions are found.

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Higher order symmetries and quantum gravitational anomalies

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Higher order symmetries are investigated in a covariant Hamiltonian formulation. The intimate relation between Killing-Yano tensors and non-standard supersymmetries is pointed out. The quantum gravitational anomalies are absent if the hidden symmetries are associated with Killing-Yano tensors. However conformal Killing tensors are sources of gravitational anomalies even if they are square of conformal Killing-Yano tensors.

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Recursion operators of difference equations

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In this talk, we'll present recursion operators for the Viallet and all of the Adler– Bobenko–Suris equations. We show that the recursion operators can indeed generate infinite hierarchies of local symmetries and conservation laws of difference equations. We also discuss the hierarchies of co-symmetries and the corresponding co-recursion operators.

Lie point symmetries of differential–difference equations

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We present an algorithm for determining the Lie point symmetries of differential equations on fixed non transforming lattices, i.e. equations involving both continuous and discrete independent variables. The symmetries of a specific integrable discretization of the Krichever-Novikov equation, the Toda lattice and Toda field theory are presented as examples of the general method. The contribution is based on joint work with D.Levi and R.Yamilov.

Lagrangian formulation of multidimensionally consistent equations II: From discrete equations to generating PDE's

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The notion of multidimensional consistency as a key integrability property is well understood; it implies that we are dealing with infinite hierarchies of compatible equations rather than with one single equation. On the discrete level, Lobb and Nijhoff have shown recently that this property is also reflected to the Lagrangian formulation of such equations. In this talk, we will show that similar constructions are possible for systems of differential equations which can be derived from the ABS equations. More precisely, starting from the latter and their master symmetries, one can derive systems of differential equations which are multidimensionally consistent in their own right. On the Lagrangian level, we establish the latter property by constructing Lagrangian multiforms which obey a closure relation; in the example we consider, they are closed two-forms.

Poisson Ideals in Cluster Algebras

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One can associate to any cluster algebra A a family of compatible Poisson structures and toric actions. In this talk we will investigate ideals of A which are stable under the Poisson brackets and the torus actions. We will show that there are only finitely many such ideals and how to obtain them combinatorially from any one cluster seed. We will then discuss applications to the theories of totally non-negative varieties, prime ideals in quantized coordinate rings and stratifications of flag varieties.

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